Ricardian Trade Models

Part I: Dornbusch, Fischer, Samuelson (1977)

ECON 871

Gameplan

Most trade papers you read nowadays will have theoretical foundations in either:

- Melitz (2003), which you covered with Kim.
  - Models with heterogeneous firms and imperfect competition.
  - Trade is generated by increasing returns to scale in production.
- Eaton and Kortum (2002)
  - Ricardian model.
  - Perfect competition.
  - Trade is generated by differences in technology (productivity) across countries.

We're going to build up to EK, starting with Dornbusch, Fischer and Samuelson (1977).

A Little History of Thought

In 1817, David Ricardo provided a mathematical example showing that countries could gain from trade by exploiting differences in their ability to produce different goods.

- Two countries do better by specializing in goods and trading, even when one country has **absolute advantage**.
- Usually taught to undergrads as a way to demonstrate the gains from trade, but then "put back in the attic."[1]
- **Problem:** The Ricardian model is not tractable to solve with many goods and/or many countries.
- As a result, theoretical/quantitative literature moved toward other driving forces of trade:
  - Differences in factor endowments (Heckscher-Ohlin)
  - Increasing returns (Krugman, Melitz)

[1] Eaton and Kortum, 2012

Ricardian Revival

A couple of theoretical innovations brought about a revival of the Ricardian model.

- (TODAY) Dornbusch, Fischer, Samuelson (DFS) (1977)—two countries, many goods.
  - **Key Innovation:** Model a continuum of goods.
  - Also introduce trade costs.
  - **Limitation:** Still only two countries.
- (WEDNESDAY) Eaton and Kortum (EK) (2002)—many countries, many goods.
  - **Key Innovation:** Model productivity in different countries as realizations of random variables.

Ricardian Model with a Continuum of Goods

DFS is based on the **Ricardian model**.

- Learn this in ECON 1 or undergrad trade.
- Trade and specialization patterns are determined by countries having *different technologies or productivities*.

Key Features:

- **Absolute Advantage:** Countries have productivity in producing certain goods.
- **Comparative Advantage:** Lower *opportunity cost* of producing some goods.
- Comparative rather than absolute advantage **determines trade patterns**.

Main Drawback: the Ricardian model is not tractable to solve with a large number of goods and/or countries.

DFS: Environment and Endowments[2]

The **breakthrough** of DFS was to model a **continuum of sectors**, which makes the characterization of the equilibrium fairly simple.

- Two countries: Home (H) and Foreign (F).
- Continuum of homogeneous goods, $z \in [0, 1]$.
- Labor is the only factor of production:
  - Country $i \in [H, F]$ is populated by L_i workers.
  - Each worker is paid a wage, w_i.
- Perfect competition + constant returns to scale.
- Costless trade (for now).

[2] A portion of these notes are based on lecture slides from Elhanan Helpman.

DFS: Demand

There is a representative consumer in each country $j \in [H, F]$ that has Cobb-Douglas preferences over goods:

$$U_j(q) = \int_0^1 b(z) \ln q(z) dz$$

- z indexes the good.
- $b(z)$ is the share of expenditure on good z.
- Assume that $\int_0^1 b(z) dz = 1$.

DFS: Demand

Utility maximization with Cobb-Douglas preferences implies:

$$p_H(z)q_H(z) = b(z)Y_H$$
$$p_F(z)q_F(z) = b(z)Y_F$$

- Where $p_i(z)q_i(z)$ is expenditure on good z in country i.
- $Y_i = w_i L_i$ is total income in country i.

DFS: Supply

Technology: Assume that each good z has a unit labor requirement $a_i(z)$ in country i.

- For a continuum of goods, we can define a function:

$$A(z) \equiv \frac{a_F(z)}{a_H(z)}, \quad A'(z) < 0$$

- This z, increases, H's comparative advantage decreases.
- Or, H has a comparative advantage in low-z goods, while F has a comparative advantage in high-z goods.

DFS: Supply

The cost of producing good z is given by:

$$\begin{aligned} &w_H \times a_H(z) \;\text{ in country } H\text{, and} \\ &w_F \times a_F(z) \;\text{ in country } F \end{aligned}$$

Simple Production Rule: Good z will be produced in H if:

$$w_H \times a_H(z) \leq w_F \times a_F(z) \iff A(z) > \frac{w_F}{w_H}$$

And, similarly, good z will be produced in F if:

$$a_H(z)w_H > a_F(z)w_F \iff A(z) < \frac{w_F}{w_H}$$

DFS: Equilibrium

Two objects will characterize the equilibrium:

1. Relative wages:
$$\omega = \frac{w_H}{w_F}$$

2. Cut-off good, $\bar{z}$, such that:
   - H produces every good $z \leq \bar{z}$.
   - F produces every good $z \geq \bar{z}$.

DFS: Equilibrium

Two unknowns, so we need two equations.

1. From the **production rule**, we know that

$$A(\bar{z}) = \frac{w_H}{w_F} = \omega$$

2. For the second condition, impose **balanced trade**, and let $G(\bar{z} = \int_0^{\bar{z}} b(z)dz$ be the share of income spent on goods produced in H.

   Balanced trade implies:

$$\underbrace{G(\bar{z})w_F L_F}_{\text{Home exports}} = \underbrace{[1 - G(\bar{z})]\, w_H L_H}_{\text{Home imports}}$$

   Rearranging, this is:

$$\omega = \frac{G(\bar{z})}{1 - G(\bar{z})} \frac{L_F}{L_H} \equiv B(\bar{z})$$

DFS

So, now we have a system of two equations ($A(\bar{z})$ and $B(\bar{z})$) in two unknowns—$\bar{z}$ and ω.

$$\omega = A(\bar{z}) \tag{1}$$

$$\omega = \frac{G(\bar{z})}{1 - G(\bar{z})} \frac{L_F}{L_H} = B(\bar{z}) \tag{2}$$

Notes:

- The $A(z)$ curve is monotonically decreasing in z (by design).
- On the other hand, $G'(\bar{z}) > 0$, so $B(z)$ is monotonically increasing in z.
- Also note that $B(0) = 0$ and $\lim_{z\to 1} B(z) = +\infty$.
- Hence, we have a unique equilibrium.

DFS: Equilibrium

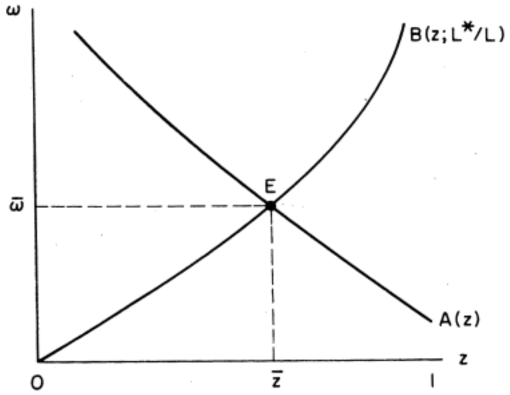

FIGURE 1

Comparative Statics: Population Growth

- **Suppose that the population in F increases**. From the point of view of H, you can interpret this as trade integration with a large country.

- $L^F \uparrow \implies B(\bar{z})$ will shift upward. (Recall: $\omega = \frac{G(\bar{z})}{1-G(\bar{z})}\frac{L^F}{L^H}$)

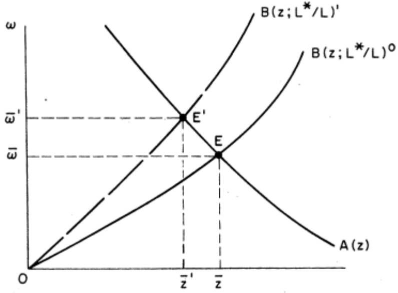

FIGURE 2

Comparative Statics: Population Growth

So, if $\frac{L^F}{L^H}$ increases:

- Home's **welfare improves**.
  - A fall in the set of goods produced ($\bar{z} \downarrow$).
  - Real income in terms of goods produced in H is constant.
  - Real income will increase in terms of imported goods.
- Foreign's **welfare worsens**.
  - Increase in the set of goods produced.
  - Real income is constant in terms of goods produced in F.
  - Real income declines in terms of goods produced in H.

Comparative Statics: Population Growth

To see this in more detail, let's normalize $w_H = 1$. Consider prices faced by consumers in H:

- Then, $Y_H' = Y_H = L_H$, by choice of numeraire.
- If good z's production *remains* in H:

$$p_H(z) = a_H(z)w_H = p_H(z)'$$

- If good z's production *remains* in F:

$$w_F' < w_F \implies p_H(z)' = w_F' a_F(z) < p_H(z)$$

- If good z's production *moves to F*:

$$w_F' a_F(z) \leq a_H(z) \implies p_H(z)' < p_H(z)$$

Welfare Changes: Intuition

- At the initial equilibrium, the increase in the foreign relative labor force creates an **excess supply of labor abroad, and an excess demand for labor at home**.

- This corresponds to a *trade surplus* in the home country. Recall, under balanced trade:

$$\underbrace{w^* L^* G(\hat{i})}_{\text{Exports from H to F}} = \underbrace{wL\left[1 - G(\hat{i})\right]}_{\text{Exports from F to H}}$$

- The increase in home country real wages **eliminates the surplus**, but also raises relative unit labor costs at home.

- The increase in relative unit labor costs in H, $\frac{w}{w^*} \uparrow$, implies a **loss of comparative advantage in marginal industries**, and thus a needed reduction in the range of commodities produced.

DFS and the Gravity Equation

The DFS model predicts a simplified version (with no trade frictions) of the **gravity equation**.

- Gravity equations in trade are a model of **bilateral trade flows** in which **size** and **distance** effects enter multiplicatively—like the law of gravity in physics.

$$\text{Newton's Law:} \qquad F_{ij} = \frac{M_i M_j}{D_{ij}^2}$$

$$\text{Gravity in Trade:} \qquad V_{od} = \frac{Y_o Y_d}{D_{od}}$$

- Workhorse for analyzing determinants of bilateral trade flows for 50+ years.
- We'll come back to this.

DFS and the Gravity Equation

Back to DFS, the trade volume can be written as:

$$2w^*L^*G(\hat{i}) = 2\frac{(wL) \times (w^*L*)}{w^*L^* + wL} = 2\frac{Y \times Y^*}{Y^W}$$

where Y and Y^* are home and foreign GDP and $Y^W = Y + Y^*$.

- This is a simplified version of the gravity equation, where trade flows depend on country size.
- It can be shown that *any model* with complete specialization, homothetic preferences, and no trade barriers delivers this prediction.

Transport Costs and Non-Traded Goods

DFS also analyze economies with **iceberg transport costs**.

- **Iceberg Cost:** To get 1 unit of good z from H to F, have to ship $\tau > 1$ units.
- H produce commodities for which domestic labor costs falls short of foreign labor costs *adjusted* for the iceberg cost:

$$\tau w_H a_H(z) \leq w_F a_F(z)$$

- Similarly, country F will produce commodities for which:

$$w_H a_H(z) \geq \tau w_F a_F(z)$$

Transport Costs and Non-Traded Goods

We can define two cut-off goods, $\underline{z}$ and $\bar{z}$:

- Define $\underline{z}$ such that $\tau w_H a_H(\bar{z}) = w_f a_f(\bar{z})$.
  Home will produce and export $z \in [0, \underline{z}]$
- Define $\bar{z}$ such that $w_H a_H(\bar{z}) = \tau w_F a_F(\bar{z})$
  Foreign will produce and export $z \in [\bar{z}, 1]$

Graphically, now there is a gap between the two $A(z)$ schedules:

- The home country produces and exports commodities to the **left** of the $\frac{A(z)}{\tau}$ schedule.
- Both countries produce commodities in the intermediate range—these are **non-tradables**.
- F produces and exports commodities to the **right** of $\tau A(z)$.

Transport Costs and Non-Traded Goods

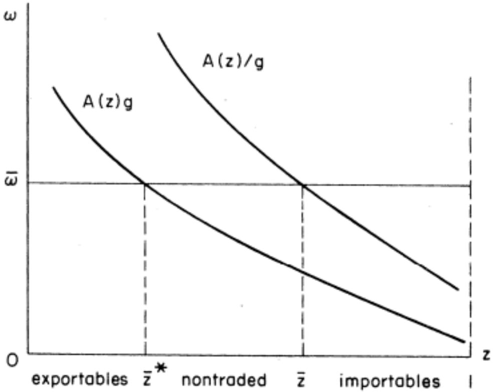

FIGURE 3

Many-Country Case

One approach to generalize DFS to many countries can be found in Costinot (2009).

- **Trick:** Put structure on the variation of the unit-labor requirements across countries and products so that it looks like the two-country world.
- Assume N countries and G goods.
- Suppose that:
  - Countries ($i = 1, ..., N$) have characteristics γ^i.
  - Goods ($g = 1, ..., G$) have characteristics σ^g.
- Let $a(\sigma, \gamma)$ denote the unit labor requirement in a sector σ and country γ.

Many-Country Case

Definition: $a(\sigma, \gamma)$ is **strictly log-submodular** if for any $\sigma > \sigma'$ and $\gamma > \gamma'$, we can write:

$$a(\sigma, \gamma) a(\sigma', \gamma') < a(\sigma, \gamma') a(\sigma', \gamma)$$

- If we assume that a is strictly positive, we can arrange this as:

$$\frac{a(\sigma, \gamma)}{a(\sigma', \gamma)} < \frac{a(\sigma, \gamma')}{a(\sigma', \gamma')}$$

- Intuitively, tihs means that high-γ countries have a comparative advantage in high-σ sectors.
- This allows us to index countries by j such that $\frac{a^{j+1}(i)}{a^{j}(i)}$ is a strictly decreasing function of i.
- If we assume there is positive demand for all products, it follows that each country will produce an internval of products with the interval of country j below that of $j+1$ etc.

Next Lecture: Eaton and Kortum (2002)

Eaton and Kortum (2002) extend DFS to a multi-good, multi-country model.

- Parametric assumption on the distribution of $a_i(z)$'s.
- Closed-form gravity equation.
- Model well suited for quantitative work.

References I

Costinot, Arnaud, "An elementary theory of comparative advantage," *Econometrica*, 2009, *77* (4), 1165–1192.

Dornbusch, Rudiger, Stanley Fischer, and Paul Anthony Samuelson, "Comparative advantage, trade, and payments in a Ricardian model with a continuum of goods," *The American Economic Review*, 1977, *67* (5), 823–839.

Eaton, Jonathan and Samuel Kortum, "Technology, geography, and trade," *Econometrica*, 2002, *70* (5), 1741–1779.

Melitz, Marc J, "The impact of trade on intra-industry reallocations and aggregate industry productivity," *econometrica*, 2003, *71* (6), 1695–1725.