

Gravity Models

ECON 871

Gravity Over Time

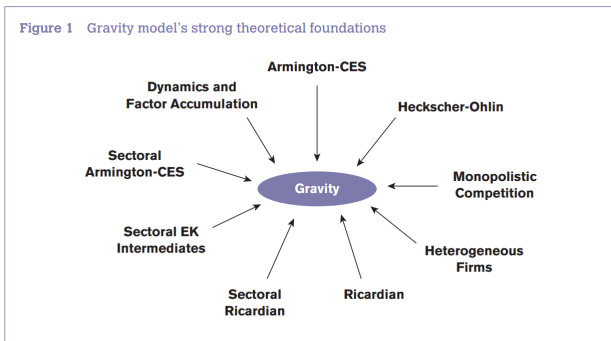
Gravity in international trade implies that countries trade in proportion to **their respective market size** (e.g., GDP) and **proximity**.¹

- ▶ Based on Newton's Law of Universal Gravitation—initial applications to economics were purely empirical.
[Ravenstein \(1885\)](#), [Tinbergen \(1962\)](#)
- ▶ [Anderson \(1979\)](#) and [Bergstrand \(1985\)](#) are among the first to offer a theoretical foundation for the gravity equation.
- ▶ Didn't really take off until [Eaton and Kortum \(2002\)](#), who derive gravity on the supply side as a Ricardian structure with intermediate goods and [Anderson and Van Wincoop \(2003\)](#) who derive gravity in an Armington-CES model.

¹Sources for these notes: [Head and Mayer \(2014\)](#) and [Yotov et al. \(2016\)](#).

Gravity Over Time

Arkolakis et al. (2012) demonstrated that a large class of models generate *isomorphic gravity equations*, which preserves the gains from trade.



SOURCE: [An Advanced Guide to Trade Policy Analysis](#)

Simple Structural Gravity Model

Consider the following model environment:

- ▶ N countries.
- ▶ Each economy produces a variety of goods (i.e., **goods are differentiated by origin**, as in Armington (1969)).
- ▶ Fixed supply of each good, Q_i , sold at factory-gate price p_i .
- ▶ Thus, the **value of domestic production** in country i is $Y_i = p_i Q_i$.
- ▶ Country i 's **aggregate expenditure** is $E_i = \phi_i Y_i$, where $\phi_i > 1$ indicates country i runs a **trade deficit** and $\phi_i < 1$ reflects a **trade surplus**. (We will take trade balance as exogenous.)

Simple Structural Gravity Model

On the demand side, consumer preferences are homothetic, identical across countries, and given by a CES utility function for country j :

$$U_j = \left[\sum_i \alpha_i^{\frac{1-\sigma}{\sigma}} c_{ij}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- ▶ $\sigma > 1$ is the elasticity of substitution among different varieties (countries).
- ▶ $\alpha_i > 0$ is an exogenous taste parameter.
- ▶ c_{ij} denotes consumption of varieties from country i in country j .

Simple Structural Gravity Model

Consumers maximize utility subject to a standard budget constraint:

$$\sum_i p_{ij} c_{ij} = E_j$$

- ▶ Here, $p_{ij} = p_i t_{ij}$ are *delivered prices* from i to j .
 - ▶ p_i are factory-gate prices in the country of origin, i .
 - ▶ $t_{ij} \geq 1$ are bilateral trade costs between i and j .
- ▶ Customary to think of the t_{ij} 's as iceberg trade costs.

Simple Structural Gravity Model

Solving the consumer's optimization problem yields the expenditures on goods shipped from origin i to destination j as:

$$X_{ij} = \left[\frac{\alpha_i p_i t_{ij}}{P_j} \right]^{1-\sigma} E_j$$

- ▶ X_{ij} are trade flows from exporter i to destination j .
- ▶ P_j is the CES price index in j :

$$P_j = \left[\sum_i (\alpha_i p_i t_{ij})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

Simple Structural Gravity Model

From the last slide, we can see that expenditure in j on goods from i :

$$X_{ij} = \left[\frac{\alpha_i p_i t_{ij}}{P_j} \right]^{1-\sigma} E_j$$

- ▶ Are **proportional to total expenditure** in the destination, E_j .
- ▶ **Inversely related to the (delivered) prices** from i to j ,
 $p_{ij} = p_i t_{ij}$.
 - ▶ Depends on factory-gate prices and bilateral trade costs.
- ▶ Directly related to the CES price aggregator, P_j , reflecting **substitution effects** across varieties from different countries.
- ▶ Contingent on the **elasticity of substitution** σ_i —higher elasticity of substitution will magnify trade diversion effects.

Simple Structural Gravity Model

Last step is to impose market clearing conditions:

$$Y_i = \sum_j \left(\frac{\alpha_i p_i t_{ij}}{P_j} \right)^{1-\sigma} E_j$$

- ▶ At *delivered prices*, the value of output in country i should be equal to the total expenditure on this country's variety in all countries (including i)

Defining $Y \equiv \sum_i Y_i$, dividing by Y and rearranging, we have:

$$(\alpha_i p_i)^{1-\sigma} = \frac{Y_i/Y}{\sum_j \left(\frac{t_{ij}}{P_j} \right)^{1-\sigma} (E_j/Y)}$$

Simple Structural Gravity Model

From the last slide:

$$(\alpha_i p_i)^{1-\sigma} = \frac{Y_i/Y}{\sum_j \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} (E_j/Y)}$$

Following Anderson and van Wincoop (2003), define the term in the denominator:

$$\Pi_i^{1-\sigma} \equiv \sum_j (t_{ij}/P_j)^{1-\sigma} E_j/Y$$

Substitute back in:

$$(\alpha_i p_i)^{1-\sigma} = \frac{Y_i/Y}{\Pi_i^{1-\sigma}}$$

Simple Structural Gravity Model

Now substitute in for $(\alpha_i p_i)^{1-\sigma}$ in our expression for X_{ij}

$$X_{ij} = \left[\frac{\alpha_i p_i t_{ij}}{P_j} \right]^{1-\sigma} E_j \rightarrow \boxed{X_{ij} = \frac{Y_i E_j}{Y} \left[\frac{t_{ij}}{\Pi_i P_j} \right]^{1-\sigma}}$$

The boxed term is the **theoretical gravity equation**. Two components:

1. A **size term**: $Y_i E_j / Y$ representing the hypothetical level of frictionless trade between i and j .
2. A **trade cost term** $(t_{ij} / \Pi_i P_j)^{1-\sigma}$, which captures the total effects of trade costs that drive a wedge between realized and frictionless trade.

Simple Structural Gravity Model

The **size term**: $Y_j E_j / Y$ provides some info on the relationship between country size and bilateral trade flows:

- ▶ Large producers will export more to all destinations.
- ▶ Bigger markets will import more from all sources.
- ▶ Trade flows between i and j will be larger the more similar in size the trading partners are.

Simple Structural Gravity Model

The **trade cost term** $(t_{ij}/\Pi_i P_j)^{1-\sigma}$, has several components:

- ▶ Bilateral trade cost between i and j , t_{ij} is typically approximated by geographic and trade policy variables (distance, tariffs, RTAs, etc.).
- ▶ Structural term P_j is an *inward multilateral resistance* term representing importer j 's ease of market access.
- ▶ Structural term Π_i is *outward multilateral resistance*—exporter i 's ease of market access.

Note the Resemblance

Let T_{ij}^{θ} be the inverse of the trade cost term and define $\tilde{G} \equiv 1/Y$.
Our gravity equation becomes:

$$X_{ij} = \tilde{G} \frac{Y_i E_j}{T_{ij}^{\sigma}}$$

Compare this to Newton's Law of Universal Gravitation:

$$F_{ij} = G \frac{M_i M_j}{D_{ij}^2}$$

Isomorphic to EK

We just derived gravity using a demand-side approach, but it is isomorphic to the supply side approach in EK. Recall, from EK, the probability that country i sells a good to j is:

$$\pi_{ij} = \frac{T_i (c_i t_{ij})^{-\theta}}{\Phi_j}$$

And the fraction of goods that j buys from i , π_{ij} , is also the fraction of its expenditures spent on goods from i :

$$X_{ij} = \frac{T_i (c_i t_{ij})^{-\theta}}{\Phi_j} E_j = \frac{T_i (c_i t_{ij})^{-\theta}}{\sum_{k=1}^N T_k (c_k t_{kj})^{-\theta}} E_j$$

Isomorphic to EK

Imposing market clearing conditions and substituting in for $T_i c_i^{-\theta}$, we get:

$$X_{ij} = \frac{t_{ij}^{-\theta}}{\Phi_j \left(\sum_{j=1}^N \frac{t_{ij}^{-\theta}}{\Phi_j} E_j \right)} Y_i E_j$$

Now, replacing Φ_j using $P_j = \gamma \Phi_j^{-\frac{1}{\theta}}$, this becomes:

$$X_{ij} = \frac{t_{ij}^{-\theta}}{\gamma^\theta P_j^{-\theta} \left(\sum_{j=1}^N \frac{t_{ij}^{-\theta}}{P_j^{-\theta}} E_j \right)} Y_i E_j$$

Isomorphic to EK

Finally, define Π_i as:

$$\Pi_i = \left(\sum_{j=1}^N \left(\frac{t_{ij}}{P_j} \right)^{-\theta} \frac{E_j}{Y} \right)^{-\frac{1}{\theta}}$$

where $Y \equiv \sum_j Y_j$. And note that P_j can be expressed as:

$$P_j = \left(\sum_{i=1}^N \left(\frac{t_{ij}}{\Pi_i} \right)^{-\theta} \frac{Y_i}{Y} \right)^{-\frac{1}{\theta}}$$

Isomorphic to EK

Now we can write our expression for X_{ij} as follows:

$$X_{ij} = \frac{Y_i E_j}{Y} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{-\theta}$$

This is the same equation we derived in the Arminton-type model, but we have replaced the elasticity of substitution, $(1 - \sigma)$, with $-\theta$ —the Fréchet parameter governing variation within the distribution.

Gravity Estimation

Typical to log-linearize the structural gravity equation and add an error term $\varepsilon_{ij,t}$:

$$\ln X_{ij,t} = \ln E_{j,t} + \ln Y_{i,t} - \ln Y_t - \theta \ln t_{ij,t} - (1 - \sigma) \ln P_{j,t} - (1 - \sigma) \ln \Pi_{i,t} + \varepsilon_{ij,t}$$

- ▶ The terms in blue are exporter characteristics.
- ▶ The terms in red are importer characteristics.
- ▶ **Problem:** $P_{j,t}$ and $\Pi_{i,t}$ are structural parameters, with no observable counterparts.
- ▶ Two typical approaches:
 - ▶ Fixed Effects Estimation
 - ▶ Ratio-Type Estimation

Fixed-Effects Estimation

$$\ln X_{ij,t} = \ln E_{j,t} - (1 - \sigma) \ln P_{j,t} + \ln Y_{i,t} - (1 - \sigma) \ln \Pi_{i,t} - \ln Y_t - \theta \ln t_{ij,t} + \varepsilon_{ij,t}$$

Basic Idea: We can't observe $P_{j,t}$ and $\Pi_{i,t}$, but we can replace them with exporter-time and importer-time fixed effects.

- ▶ Several benefits to this approach:
 - ▶ Can get a consistent estimate of θ , which is usually the parameter of interest.
 - ▶ Don't have to make many strong structural assumptions.
- ▶ Importantly, the FEs will also absorb the rest of the blue/red terms (country size) and other unobservable country-specific characteristics.

$$\ln X_{ij,t} = \ln Y_t + \ln S_j + \ln M_i - \theta \ln t_{ij,t} + \varepsilon_{ij,t}$$

Fixed-Effects Estimation

There are potentially interesting trade determinants that can no longer be identified if we use importer/exporter FEs. Notably:

1. Anything that affects exporters' propensity to export to all destinations.
 - ▶ e.g., hosting an Olympics.
2. Variables that affect imports without regard to country of origin.
 - ▶ e.g., country-level average tariff rates.

There are possible ways to estimate the effects of these *monadic* variables of interest even with FEs.

Fixed-Effects Estimation

More generally, suppose we want to estimate the following (changing notation slightly):

$$\ln X_{ni} = \alpha_i + \beta V_i + \gamma_n + \delta D_{ni} + \varepsilon_{ni}$$

- ▶ V_i is a monadic variable of interest (e.g., cost or quality of exports from country i).
- ▶ D_{ni} are the “dyadic” controls (trade frictions—e.g., distance, RTAs).
- ▶ α_i are the other i – level determinants of exports (other exporter FEs).
- ▶ γ_n are the importer FEs.

A few approaches to estimating β .

Fixed-Effects Estimation

$$\ln X_{ni} = \alpha_j + \beta V_i + \gamma_n + \delta D_{ni} + \varepsilon_{ni}$$

One-Step Approach: Combine α_j and ε_{ni} as the error term.

- ▶ Even if α_j is uncorrelated with V_i (which is unlikely), the error terms for the same exporter will be correlated.
- ▶ This will result in downward-biased standard errors of β unless standard errors are **clustered by exporter**.

Fixed-Effects Estimation

$$\ln X_{ni} = \alpha_i + \beta V_i + \gamma_n + \delta D_{ni} + \varepsilon_{ni}$$

Two-Step Estimator: Basic idea is to do the following:

1. First, estimate the two-way fixed effects version:

$$\ln X_{ni} = S_i + \gamma_n + \delta D_{ni} + \varepsilon_{ni}$$

2. Then, regress $\ln \hat{S}_i$ on V_i .
 - ▶ EK 2002 do a version of this to estimate θ (not their baseline).
 - ▶ Gives an unbiased estimate of $\hat{\beta}$.

Fixed-Effects Estimation

Lastly, we can get a one-step estimator for both $\hat{\beta}$ and $\hat{\delta}$ by modeling α_j as:

$$\alpha_j = \alpha_0 + \alpha_1 C_j + \alpha_2 \bar{D}_j + \psi_j$$

Where:

- ▶ C_j are i -specific controls.
- ▶ \bar{D}_j are average characteristics of each exporter:
$$\bar{D}_j = \frac{1}{N} \sum_n D_{ni}$$
- ▶ An error term.

Subbing this in to our original estimating equation:

$$\ln X_{ni} = \alpha_0 + \alpha_1 C_i + \beta V_i + \gamma_n + \delta D_{ni} + \alpha_2 \bar{D}_i + (\psi_i + \varepsilon_{ni})$$

- ▶ Again, we have to cluster standard errors at the i -level because of the ψ_i .

Ratio-Type Estimation

Another solution to estimation of the θ 's involves using the multiplicative structure of the gravity model to eliminate the monadic terms.

Let's rewrite the gravity equation one more time as:

$$X_{ni} = GS_i M_n \phi_{ni}$$

- ▶ S_i are the exporter characteristics.
- ▶ M_n are the importer characteristics.
- ▶ ϕ_{ni} are the bilateral relationship characteristics (trade frictions).
- ▶ G is a "gravitational constant." (Constant in the cross section.)

Ratio-Type Estimation

Eaton and Kortum (2002) normalize bilateral trade flows, X_{ni} by own-trade flows:

$$\frac{X_{ni}}{X_{nn}} = \frac{GS_i M_n \phi_{ni}}{GS_n M_n \phi_{nn}} = \left(\frac{S_i}{S_n} \right) \left(\frac{\phi_{ni}}{\phi_{nn}} \right)$$

- ▶ This removes the importer FE, M_n .
- ▶ Still have to measure S terms, presumably with substantial measurement error.

Head and Ries (2001) propose a simple solution to cancel out the exporter terms as well—multiply the above by $X_{in}X_{ji}$:

$$\left(\frac{X_{ni}}{X_{nn}} \right) \left(\frac{X_{in}}{X_{ji}} \right) = \left(\frac{S_i}{S_n} \right) \left(\frac{\phi_{ni}}{\phi_{nn}} \right) \times \left(\frac{S_n}{S_i} \right) \left(\frac{\phi_{in}}{\phi_{ji}} \right)$$

Ratio-Type Estimation

If we make the following two assumptions:

- ▶ Symmetry in bilateral trade costs: $\phi_{ni} = \phi_{in}$
- ▶ Frictionless trade inside countries $\phi_{nn} = \phi_{ii} = 1$

We end up with a simple index that Eaton et. al. (2011) call the Head-Ries Index (HRI):

$$\hat{\phi}_{ni} = \sqrt{\frac{X_{ni}X_{in}}{X_{nn}X_{ii}}}$$

Problem:

- ▶ Need a measure of own-country trade flows, X_{ij} 's.
- ▶ Can be proxied using production minus total exports, but this often generates negative observations related to measurement issues.

Ratio-Type Estimation

Caliendo and Parro (2015) estimate the trade cost elasticity using tariff data and exploiting asymmetries in protectionism as an identification strategy.

Suppose trade costs can be written:

$$\phi_{ni} = \left[(1 + \tau_{ni}) d_{ni}^{\delta} \right]^{\epsilon}$$

Where $d_{ni} = d_{in}$ captures all symmetric trade costs (e.g., distance) in $X_{ni} = GS_i M_n \phi_{ni}$. We can introduce a third country, h , and then multiply X_{ni}/X_{nh} , X_{ih}/X_{hi} , and X_{hn}/X_{in} :

$$\frac{X_{ni} X_{ih} X_{hn}}{X_{nh} X_{hi} X_{in}} = \left(\frac{(1 + t_{ni})(1 + t_{ih})(1 + t_{hn})}{(1 + t_{nh})(1 + t_{hi})(1 + t_{in})} \right)^{\epsilon}$$

We can estimate this with bilateral tariff data.

References I

- Anderson, James E**, “A theoretical foundation for the gravity equation,” *The American economic review*, 1979, 69 (1), 106–116.
- **and Eric Van Wincoop**, “Gravity with gravitas: A solution to the border puzzle,” *American economic review*, 2003, 93 (1), 170–192.
- Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare**, “New trade models, same old gains?,” *American Economic Review*, 2012, 102 (1), 94–130.
- Bergstrand, Jeffrey H**, “The gravity equation in international trade: some microeconomic foundations and empirical evidence,” *The review of economics and statistics*, 1985, pp. 474–481.
- Caliendo, Lorenzo and Fernando Parro**, “Estimates of the Trade and Welfare Effects of NAFTA,” *The Review of Economic Studies*, 2015, 82 (1), 1–44.

References II

- Eaton, Jonathan and Samuel Kortum**, “Technology, geography, and trade,” *Econometrica*, 2002, 70 (5), 1741–1779.
- Head, Keith and John Ries**, “Increasing returns versus national product differentiation as an explanation for the pattern of US–Canada trade,” *American Economic Review*, 2001, 91 (4), 858–876.
- **and Thierry Mayer**, “Gravity equations: Workhorse, toolkit, and cookbook,” in “Handbook of international economics,” Vol. 4, Elsevier, 2014, pp. 131–195.
- Ravenstein, Ernst Georg**, “The laws of migration,” in “in” Royal Statistical Society 1885.
- Tinbergen, Jan**, “Shaping the world economy; suggestions for an international economic policy,” 1962.
- Yotov, Yoto V, Roberta Piermartini, Mario Larch et al.**, *An advanced guide to trade policy analysis: The structural gravity model*, WTO iLibrary, 2016.